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Optimum Cooperative Spectrum Sensing Technique for Multiuser Ultraviolet Wireless Communications

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***Abstract*—In this paper, we present a novel optimum cooperative spectrum sensing technique to mitigate multiuser interference for multiuser ultraviolet wireless communications over Málaga distributed turbulence channel. We consider the distributed decision fusion for the cooperative sensing. Based on the derived Málaga distribution, a mathematically-tractable expression for the average probability of detection is presented. An optimal voting rule is derived to minimize the average error rate. To verify this optimum voting rule, we use the energy detection technique. It is found that the formulated voting rule produces one optimal value only, which indeed confirms its optimum performance.**

***Index Terms*—Probability of detection, scattering, ultraviolet communication.**

I. Introduction

With wavelengths ranging from ultraviolet (UV) to infrared, there have been increasing efforts in developing a reliable optical wireless communication (OWC) system as it serves as a future candidate for wireless communications. OWC offers a low-cost and low-power transmission with potentially large bandwidth. [1].

Although infrared and visible lights come with inherent advantages, they are vulnerable to blockage because of their directionality. Moreover, the detection performance is also significantly limited in outdoor applications, because of solar background. A promising solution to these problems is to employ solar blind UV communication which comes with unique channel characteristics and offers non-line-of-sight (NLOS) communication links [2].

In an optical ad hoc network, to avoid the interference from other potential active transceivers, the receiver needs to periodically detect the available transmission frequencies by sensing the optical spectrum. If the detector detects the available optical signal falling into its field-of-view (FOV), a feedback signal needs to be sent to the transmitter to establish a link. Optical sensing enables the receiver to continuously monitor the channel before its access to the available spectrum. A spectrum sensing technique based on the generalized likelihood ratio test (GLRT) for optical wireless scattering communications was proposed [3]. The authors compared the GLRT with the sum-counting test and interestingly found it optimal for optical spectrum sensing. However, the optimal performance comes with higher computational complexity. In [4], a less complex spectrum sensing technique for optical scattering communication systems was reported. The technique based on sequential detection for spectrum sensing adapts itself to the present channel state. To reduce the computational complexity, the authors presented a one-term approximation of the log-likelihood ratio test. Recently, a blind spectrum sensing technique for multiuser optical scattering communications over Gamma-Gamma distributed turbulence channel was reported [5]. This technique is based on the estimation of the received SNR and noise power and is equipped with switch-and-stay diversity combining to enhance the optical spectrum sensing capability.

Over the years, many fading distribution models have been reported to characterize the turbulence over the optical scattering channel, such as the Gamma-Gamma [6] and the log-normal distribution [7]. Recently, a new generic statistical model, i.e., the Málaga distribution model, was proposed [8]. The Málaga distribution is valid for both plane and spherical wave propagation and is applicable under all range of turbulence conditions, i.e., weak, moderate and strong. In addition, it unifies most of the existing statistical models for the turbulence fading and it shows an excellent match to the experimental data. A unified fading model based on this Málaga distribution for optical scattering communications was reported for the first time [9]. It was demonstrated that the distribution effectively includes the scattering by the off-axis eddies, in addition to the eddies on the propagation axis, over a NLOS UV link, thereby permitting accurate performance evaluation of the NLOS UV communication.

In a multiuser ultraviolet wireless communication system, the cooperation between multiple receivers can improve the performance significantly, known as cooperative spectrum sensing. In this paper, we present a cooperative spectrum sensing technique with distributed signal processing for UV based scattering communication systems with each scattered path following the Málaga distribution [9] in a UV scattering channel model. The proposed scheme does not require any information about the transmitted UV signal and is robust to the dispersive scattered optical channel.

The main contributions of this work include:

- We introduce the distributed decision fusion based cooperative spectrum sensing for NLOS optical wireless scattering communication systems with each scattered path is Málaga distributed.
- Based on the derived model, a mathematically-tractable expression for the average probability of detection is presented.
- To minimize the average error rate, an optimal voting rule is applied. A hard decision fusion with majority rule is applied to exploit the maximum spectrum opportunities. The results demonstrate that for a target error rate, the proposed scheme requires fewer collaborative secondary users than the total number of users available.

II. System Model

The UV scattered beam that arrives at the photodetector depends on the link geometry and the atmospheric characteristics. A typical NLOS UV configuration is illustrated in Fig. 1. d represents the separation between the transmitter and the receiver. ψ_t and ψ_r are the transmitter and the receiver apex angles, respectively. ϕ_t is the transmit beam angle and ϕ_r denotes the receiver FOV. V_c represents the common volume.

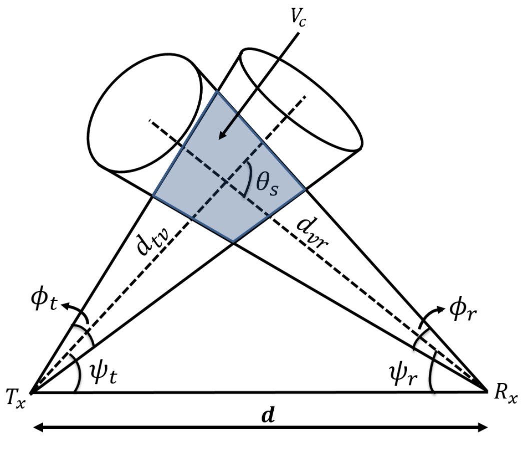

Fig. 1. NLOS UV communication link.

Atmospheric turbulence causes intensity fluctuation in the received UV signal and is considered one of the main impairment in limiting the performance of the UV communication system. Considering all these impairments, the instantaneous received signal can then be given as

$$y = Sh_t h_s P_t I + n \tag{1}$$

where S is the receiver responsivity in A/W, h_t represents the fading due to atmospheric scintillation and can be expressed as $h_t = 10^{-\frac{\sqrt{23.17k^{7/6}C_n^2\left(d_{tv}^{11/6}+d_{vr}^{11/6}\right)}}{5}}$ [6], where $k = 2\pi/\lambda$ represents the wave number and λ wavelength of the UV signal. C_n^2 denotes the refractive index coefficient of atmospheric turbulence channel. h_s in (1) denotes the attenuation term due to atmospheric scattering and can be modeled as [10]

$$h_s = \frac{A_r \alpha_s q_s \phi_t^2 \phi_r \sin(\psi_t + \psi_r)\left(12\sin^2\psi_r + \phi_r^2 \sin^2\psi_t\right)}{96 d \sin\psi_t \sin^2\psi_r \left(1-\cos\frac{\phi_t}{2}\right)\exp\left(\frac{\alpha_t d(\sin\psi_t + \sin\psi_r)}{\sin(\psi_t+\psi_r)}\right)}. \tag{2}$$

P_t is the transmit power. n is the Gaussian noise. I denotes the optical scintillation.

A. Marginal PDF of the received scattered intensity

Following the Málaga distribution, the PDF of the optical power arriving in the common volume is given by [9]

$$f_{I_v}(I_v) = A_v \sum_{k=1}^{\beta_v} a_{vk} I_v^{\left(\frac{\alpha_v+k}{2}-1\right)} K_{\alpha_v-k}\left(2\sqrt{\frac{\alpha_v\beta_v k}{\gamma_v\beta_v+\Omega'}}\right), \tag{3}$$

where

$$\begin{cases} \alpha_v = \left\{\exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}}\right)-1\right\}^{-1} \\ A_v = \frac{2\alpha_v^{\frac{\alpha_v}{2}}}{\gamma_v^{1+\frac{\alpha_v}{2}}\Gamma(\alpha_v)}\left(\frac{\gamma_v\beta_v}{\gamma_v\beta_v+\Omega'}\right)^{\beta_v+\frac{\alpha_v}{2}} \\ a_{vk} = \binom{\beta_v-1}{k-1}\frac{\left(\gamma_v\beta_v+\Omega'\right)^{1-\frac{k}{2}}}{(k-1)!}\left(\frac{\Omega'}{\gamma_v}\right)^{k-1}\left(\frac{\alpha_v}{\beta_v}\right)^{\frac{k}{2}} \end{cases}. \tag{4}$$

α_v in (4) represents the effective number of large-scale cells of the scattering process, σ_R^2 is the Rytov variance. β_v is a natural number. α_v and β_v together represent the strength of the turbulence. $\Omega' = \Omega + 2\rho b_o + 2\sqrt{2\rho b_o \Omega}cos(\phi_A - \phi_B)$, where $2b_o$ represents the average power of the total scattered components and Ω denotes the power of the unscattered components. ρ is the factor representing the power coupling between the scattered and unscattered components. ϕ_A and ϕ_B are the deterministic phases. The conditional distribution of the irradiance fluctuations I_r at the receiver can then be written as

$$f_{I_r}(I_r|I_v) = A_r \sum_{k=1}^{\beta_r} a_{rk} I_r^{\left(\frac{\alpha_r+k}{2}-1\right)} K_{\alpha_r-k}\left(2\sqrt{\frac{\alpha_r\beta_r k}{\gamma_r\beta_r+\Omega'}}\right), \tag{5}$$

where α_r, A_r, and a_{rk} can similarly be obtained as α_v, A_v, and a_{vk}. From (3) and (5) and using the identities [11, 12], the marginal PDF of the irradiance scintillation at the receiver is derived as shown in (6).

B. Marginal PDF of the instantaneous SNR

The instantaneous received SNR is given by

$$\chi = \frac{(SHP_t)^2 I_r^2}{\sigma_n^2}. \tag{7}$$

With I_v and I_r are normalized to unity, we define the average received SNR as

$$\xi = \frac{(SHP_t)^2}{\sigma_n^2}. \tag{8}$$

By applying the Jacobean transformation, the marginal PDF of the instantaneous SNR can be obtained as

$$f_\chi(\chi) = |J| f_{I_r}(I_r)\big|_{I_r=\sqrt{\frac{\chi}{\xi}}}, \tag{9}$$

$$f_{I_r}(I_r) = \frac{A_v A_r}{2} \sum_{k=1}^{\beta_v} a_{vk} (\alpha_v k) \left(\frac{\alpha_v \beta_v}{\gamma_v \beta_v + \Omega'} \right)^{-\left(\frac{\alpha_v + k}{2}\right)} \times \sum_{k=1}^{\beta_r} a_{rk} I_r^{\left(\frac{\alpha_r + k}{2} - 1\right)} K_{\alpha_r - k} \left(2 \sqrt{\frac{\alpha_r \beta_r k}{\gamma_r \beta_r + \Omega'}} \right). \tag{6}$$

where J is the Jacobean matrix. After substituting the values in (7), the closed-form expression of $f_\chi(\chi)$ is derived as (10).

III. Detction Probability over Málaga distributed Turbulence channel

Next, we provide the exact solution for the average detection probability. We consider a cooperative optical wireless scattering network with N secondary users. Each receiver listens to the optical channel and performs spectrum sensing independently. For the i^{th} user, the average probability of detection over Gaussian noise is given by [13]

$$P_{d,i} = Q_m \left(\sqrt{\frac{g\chi}{\sigma_n^2}}, \sqrt{\frac{\chi_{Th}}{\sigma_n^2}} \right), \tag{11}$$

where $Q_m(\cdot)$ is the generalized Marcum Q-function [14] with m related to the time-bandwidth product. g is some positive number and χ_{Th} is the detection threshold. Averaging over statistics of χ, the average probability of detection can be obtained as

$$P_{d,avg} = \int_0^\infty Q_m \left(\sqrt{\frac{g\chi}{\sigma_n^2}}, \sqrt{\frac{\chi_{Th}}{\sigma_n^2}} \right) f_\chi(\chi) \, d\chi \tag{12}$$

Substituting the definition of the generalized Marcum Q-function [14] and applying the Chernoff bound for Q-function [15], we obtain

$$\begin{aligned} P_{d,avg} = & \underbrace{\frac{1}{2} \int_0^\infty \left\{ 1 - \frac{g\chi + \chi_{Th} - 2\sqrt{g\chi\chi_{Th}}}{2\sigma_n^2} \right\} f_\chi(\chi) \, d\chi}_{Part\ I} \\ & + \underbrace{\int_0^\infty \left\{ \exp\left[-\left(\frac{g\chi + \chi_{Th}}{2\sigma_n^2} \right) \right] \sum_{j=1}^{m-1} \left(\sqrt{\frac{\chi_{Th}}{g\chi}} \right)^j I_j \left(\frac{\sqrt{g\chi\chi_{Th}}}{\sigma_n^2} \right) \right\} f_\chi(\chi) \, d\chi}_{Part\ II} \end{aligned}, \tag{13}$$

where $I_j(\cdot)$ denotes the modified Bessel function of the first kind and order j.

As shown, equation (13) can be divided into two parts. The integrals in *Part I* can readily be solved using the identity provided in Theorem (1).

Theorem 1. *Provided the distribution of χ, as derived in (10), the following integral can readily be obtained in a closed form as*

$$\begin{aligned} & C \int_0^\infty \chi^y f_\chi(\chi) \, d\chi = \\ & \frac{C A_v A_r}{4} \Gamma(2y + \alpha_r) (\xi)^y \\ & \times \left\{ \sum_{k=1}^{\beta_v} a_{vk} (\alpha_v k) \left(\frac{\alpha_v \beta_v}{\beta_v \gamma_v + \Omega'} \right)^{-\left(\frac{\alpha_v + k}{2}\right)} \right\} \\ & \times \left\{ \sum_{k=1}^{\beta_r} a_{rk} \Gamma(2y + k) \left(\frac{\alpha_r \beta_r}{\beta_r \gamma_r + \Omega'} \right)^{-\left(\frac{4y + \alpha_r + k}{2}\right)} \right\} \end{aligned}. \tag{14}$$

where $\Gamma(\cdot)$ represents the Gamma function.

Proof. A proof of the derivation of Theorem (1) is provided in Appendix A. □

With the general framework developed in (14), we will now provide the exact solutions for the integrals. As can be seen, *Part I* contains three integrals with $\{C_1 = 2(1 - \frac{\chi_{Th}}{2\sigma_n^2}), y_1 = 0; C_2 = (-\frac{g}{4\sigma_n^2}), y_2 = 1; C_3 = (\frac{\sqrt{\chi_{Th} g}}{2\sigma_n^2}), y_3 = 1/2\}$. Substituting the corresponding values of C and y, the integrals in *Part I* of (13) can readily be obtained in a closed form. Solving for the integral in *Part II* of (13), a closed-form mathematically-tractable analytical expression for the average detection probability is obtained as provided in (15), where $\mathbf{S}[\cdot|\cdot|\cdot]$ in (15) represents the bivariate Meijer G-function function (BMGF) as defined in [16, eq. (1)]. The proof of the integral in *Part II* of (13) is provided in Appendix B.

The probability of false alarm for the i^{th} receiver is given by

$$P_{f,i} = \frac{\Gamma\left(m, \frac{\chi_{Th}}{2}\right)}{\Gamma(m)}. \tag{16}$$

Here, without loss of generality, we have assumed that each receiver experiences identical path loss yielding $\chi_1 = \chi_2 = \cdots = \chi_N$. In addition, each receiver uses the same threshold χ_{Th}. This results in $P_{d,avg}$ and $P_{f,i}$ being independent of the receiver. Considering linear fusion, the total probabilities of detection and a false alarm in the case of cooperative spectrum sensing for an optical scattering communication system can be expressed as

$$P_{d,CSS} = \sum_{z=n}^{N} \binom{N}{z} P_{d,avg}^z \left(1 - P_{d,avg}\right)^{N-1}, \tag{17}$$

and

$$P_{f,CSS} = \sum_{z=n}^{N} \binom{N}{z} P_f^z \left(1 - P_f\right)^{N-1}. \tag{18}$$

$$f_{\chi}(\chi) = \frac{A_v A_r}{4} \sum_{k=1}^{\beta_v} a_{vk} (\alpha_v k) \left(\frac{\alpha_v \beta_v}{\gamma_v \beta_v + \Omega'} \right)^{-\left(\frac{\alpha_v + k}{2}\right)} \times \sum_{k=1}^{\beta_r} a_{rk} \chi^{\left(\frac{\alpha_r + k}{4} - 1\right)} \left(\frac{1}{\xi} \right)^{\left(\frac{\alpha_r + k}{4}\right)} K_{\alpha_r - k} \left(2 \sqrt{\frac{\alpha_r \beta_r}{\gamma_r \beta_r + \Omega'}} \sqrt{\frac{\chi}{\xi}} \right). \tag{10}$$

$$\begin{aligned} P_{d,avg} = {} & \frac{C A_v A_r}{4} \left(\sum_{k=1}^{\beta_v} a_{vk} (\alpha_v k) \left(\frac{\alpha_v \beta_v}{\beta_v \gamma_v + \Omega'} \right)^{-\left(\frac{\alpha_v + k}{2}\right)} \right) \times \Bigg\{ \frac{1}{2} \left(1 - \frac{\chi_{Th}}{2\sigma_n^2} \right) \Gamma(\alpha_r) \left(\sum_{k=1}^{\beta_r} a_{rk} \Gamma(k) \left(\frac{\alpha_r \beta_r}{\beta_r \gamma_r + \Omega'} \right)^{-\left(\frac{\alpha_r + k}{2}\right)} \right) \\ & - \left(\frac{g}{4\sigma_n^2} \right) \Gamma(2 + \alpha_r) \xi \left(\sum_{k=1}^{\beta_r} a_{rk} \Gamma(2 + k) \left(\frac{\alpha_r \beta_r}{\beta_r \gamma_r + \Omega'} \right)^{-\left(\frac{\alpha_r + k + 4}{2}\right)} \right) \\ & + \left(\frac{\sqrt{g \chi_{Th}}}{2\sigma_n^2} \right) \Gamma(1 + \alpha_r) \sqrt{\xi} \left(\sum_{k=1}^{\beta_r} a_{rk} \Gamma(1 + k) \left(\frac{\alpha_r \beta_r}{\beta_r \gamma_r + \Omega'} \right)^{-\left(\frac{\alpha_r + k + 2}{2}\right)} \right) + \exp\left(-\frac{\chi_{Th}}{\sigma_n^2} \right) \sum_{j=1}^{m-1} \left(\frac{\chi_{Th}}{g} \right)^j \\ & \times \sum_{k=1}^{\beta_r} a_{rk} \left(\frac{1}{\xi} \right)^{\left(\frac{\alpha_r + k}{4}\right)} \sum_{q=0}^{\infty} \left(\frac{\sqrt{g \chi_{Th}}}{\sigma_n^2} \right)^q \left(\frac{2}{\pi} \right)^{\frac{1}{4}} \mathbf{S} \left[\begin{array}{c|c|c} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{bmatrix} & \begin{array}{c} \frac{\alpha_r + k + 2q - 4j}{2}; - \\ \frac{1}{2}; j, -j \\ -; \frac{\alpha_r - k}{2}, -\frac{\alpha_r - k}{2} \end{array} & \begin{array}{c} 2\sqrt{\frac{\chi_{Th}}{\sigma_n^2}} \\ \left(\frac{\alpha_r \beta_r}{\beta_r \gamma_r + \Omega'} \right) \sqrt{\frac{\sigma_n^2}{g \xi}} \end{array} \\ \begin{pmatrix} 1,1 \\ 0,1 \end{pmatrix} & & \\ \begin{pmatrix} 2,0 \\ -2,2 \end{pmatrix} & & \end{array} \right] \Bigg\} \end{aligned} \tag{15}$$

The total average error rate can then readily be formulated as

$$P_{error} = 1 + P_{f,CCS} - P_{d,CCS}. \tag{19}$$

In the cooperative sensing, each cooperative receiver makes local decision, which is then fused at the center for final decision. For a given number of secondary receivers N, we use n–out–of–N voting rule where n is an integer. The optimal value of n for the minimum error rate P_{error} can then readily by obtained as[17]

$$n_{opt} = \min \left(N, \left\lceil \frac{N}{1 + \left(\frac{\ln\left(\frac{P_f}{P_{d,avg}} \right)}{\ln\left(\frac{1 - P_{d,avg}}{1 - P_f} \right)} \right)} \right\rceil \right), \tag{20}$$

where $\lceil \cdot \rceil$ is the ceiling function.

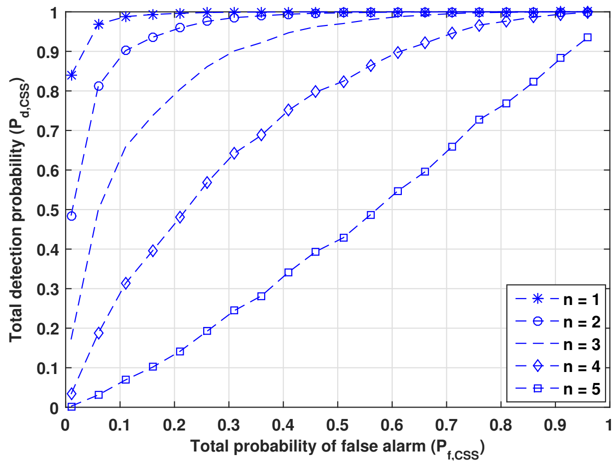

Fig. 2. ROC curve for cooperative spectrum sensing over Málaga distributed optical scattering communications (N = 5).

IV. Simulation Results and Analysis

We quantify the system performance by depicting the operating characteristics at the receiver for NLOS scattering optical communication systems. For the sake of brevity, we set $\{\alpha_v, \beta_v\} = \{\alpha_r, \beta_r\} = \{2.2814, 33\}$, representing a strong turbulence regime. Ω is set to 1.33 and b_0 is equal to 0.4231. ρ is set to 0.84 [8]. $\phi_A - \phi_B$ is set to $\pi/6$ radian. g is set to 2. About 50 terms was needed for summation to achieve accuracy at the 5th significant digit.

Figure 2 illustrates the receiver operating characteristics for different voting rules. The sample size L is set to 500. The average SNR ξ is set to 15 dB. It can be seen from the result that under the fixed target probability, $n = 1$ tends to be optimal. It should be noted that it is very difficult to obtain the target $P_{f,CSS}$ in the proposed system if there exists noise uncertainty. In addition, if the received samples are highly correlated, the performance will not be optimal.

The total error rate P_{error} relative to the detection threshold is illustrated in Fig. 3. The average SNR ξ is set to 15 dB. It can be seen from the result that in a network of 5 users, the optimal voting rule is found to be 3. Also, for high detection threshold values, the performance improves with a higher value of n.

The optimal voting rule required for a given target probability of false alarm over various received SNR values is illustrated in Fig. 4. The detection threshold χ_{Th} is set to 50.

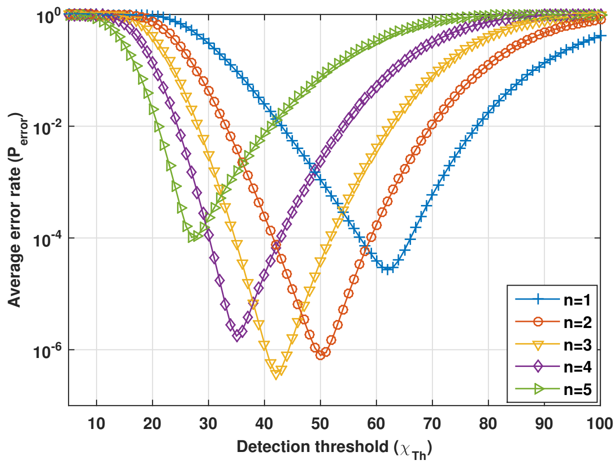

Fig. 3. Total error rate relative to the detection threshold over Málaga distributed optical scattering communications (N = 5).

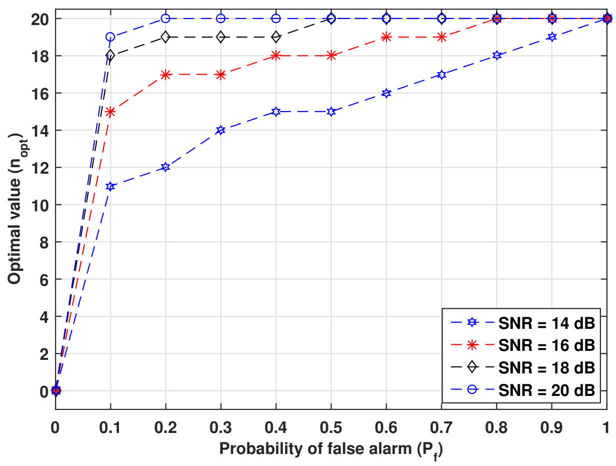

Fig. 4. Optimal voting rule against the target probability of false alarm for Málaga distributed optical scattering communications (N = 20 and $\chi_{Th} = 50$).

It is interesting to note that over the entire examined range of probability of false alarm, n_{opt} reduces with decreasing target P_f.

The optimal voting rule required against the detection threshold over various received SNR ranges is illustrated in Fig. 5. The detection threshold χ_{Th} is set to 50. It should be noted that as the detection threshold increases, the optimal voting rule reduces. In addition, for a given χ_{Th}, n_{opt} increases with increasing SNRs.

V. Conclusion

This paper has presented the cooperative spectrum sensing with distributed fusion in multiuser optical ad hoc networks with scattering communication. We have derived an exact closed-form expression for the average detection probability with each scattered path considered Málaga distributed. To minimize the total error rate, this paper has also focused on finding the robust voting rule at the fusion center based on local sensing decisions received from the secondary users. The optimal choice of voting rule is found

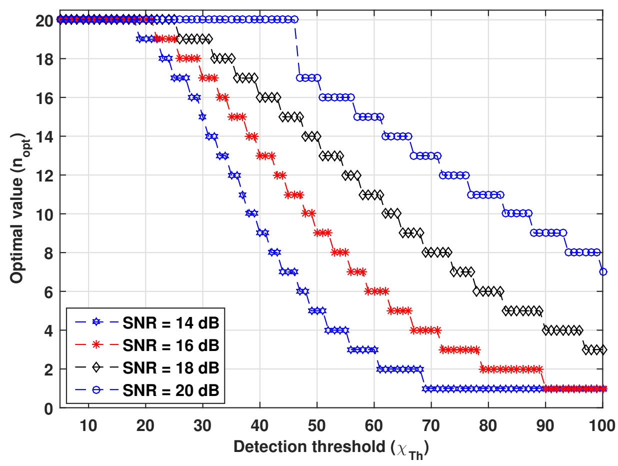

Fig. 5. Optimal voting rule against the detection threshold for Málaga distributed optical scattering communications (N = 20).

to be $N/2$, where N is the number of users. The analysis of cooperative spectrum sensing for optical scattering communications is reliable and timely for emerging applications.

Appendix A
Proof of Theorem (1)

Substituting for $f_\chi(\chi)$ from (10) and using the identity [11, eq. (14)] with change of variable χ to ϕ^2, the integral in (14) can be written as

$$
\begin{aligned}
&C\int_0^\infty \chi^y f_\chi(\chi)\,d\chi = \\
&\frac{CA_vA_r}{4}\left\{\sum_{k=1}^{\beta_v} a_{vk}(\alpha_v k)\left(\frac{\alpha_v\beta_v}{\beta_v\gamma_v+\Omega'}\right)^{-\left(\frac{\alpha_v+k}{2}\right)}\right\} \\
&\times\left\{\sum_{k=1}^{\beta_r} a_{rk}\left(\frac{1}{\xi}\right)^{\left(\frac{\alpha_r+k}{4}\right)}\left(\int_0^\infty \phi^{\left(\frac{4y+\alpha_r+k}{2}-1\right)}\right.\right. \\
&\left.\left. G_{2,0}^{0,2}\left[\left(\frac{\alpha_r\beta_r}{\beta_r\gamma_r+\Omega'}\right)\frac{\phi}{\sqrt{\xi}}\,\middle|\,\begin{matrix} - \\ \frac{\alpha_r+k}{2},-\frac{\alpha_r+k}{2}\end{matrix}\right]d\phi\right)\right\}
\end{aligned}
\quad . \qquad (21)
$$

Using the identity [11, eq. (24)], the integral in (21) can then readily be simplified into closed-form as provided in (14).

Appendix B
Proof of the integral in *Part II* of (13)

Substituting for $f_\chi(\chi)$ from (10) and using the identities [18, eq. (03.02.26.0009.01)] and [11, eq. (14)], the integral, after simple mathematical manipulations, can be written as

$$\begin{aligned}
&\int_0^\infty \left\{ \exp\left[-\left(\frac{g\chi+\chi_{Th}}{2\sigma_n^2}\right)\right] \sum_{j=1}^{m-1} \left(\sqrt{\frac{\chi_{Th}}{g\chi}}\right)^j \right.\\
&\left. I_j\left(\frac{\sqrt{g\chi\chi_{Th}}}{\sigma_n^2}\right)\right\} f_\chi(\chi)\,d\chi \Bigg\} = \\
&\frac{CA_vA_r}{4}\left\{\sum_{k=1}^{\beta_v} a_{vk}\left(\alpha_v k\right)\left(\frac{\alpha_v\beta_v}{\beta_v\gamma_v+\Omega'}\right)^{-\left(\frac{\alpha_v+k}{2}\right)}\right\} \\
&\times \exp\left(-\frac{\chi_{Th}}{2\sigma_n^2}\right) \\
&\times \frac{1}{2\sqrt{\pi}} \sum_{j=1}^{m-1}\left\{\left(\frac{\chi_{Th}}{g}\right)^j \sum_{k=1}^{\beta_r}\left[a_{rk}\left(\frac{1}{\xi}\right)^{\left(\frac{\alpha_r+k}{4}\right)}\right.\right. \\
&\times \sum_{q=0}^{\infty}\left(\left(\frac{\sqrt{g\chi_{Th}}}{\sigma_n^2}\right)^q \int_0^\infty \exp\left(-\frac{g\chi}{2\sigma_n^2}\right)\chi^{\frac{\alpha_r+k+2q-4j}{4}}\right. \\
&\times G_{1,2}^{1,1}\left[\frac{2\sqrt{g\chi_{Th}\chi}}{\sigma_n^2}\,\middle|\, \begin{matrix}\frac{1}{2}\\ j,-j\end{matrix}\right] \\
&\left.\left.\left.\times G_{2,0}^{0,2}\left[\left(\frac{\alpha_r\beta_r}{\beta_r\gamma_r+\Omega'}\right)\sqrt{\frac{\chi}{\xi}}\,\middle|\, \begin{matrix}-\\ \frac{\alpha_r+k}{2},-\frac{\alpha_r+k}{2}\end{matrix}\right] d\chi\right)\right]\right\}
\end{aligned} \tag{22}$$

The product of the two Meijer G-functions in (22) can readily be solved using the identity [19, eq. (6)] which yields the form as shown in (23).

$$\begin{aligned}
&\int_0^\infty \left\{ \exp\left[-\left(\frac{g\chi+\chi_{Th}}{2\sigma_n^2}\right)\right] \sum_{j=1}^{m-1} \left(\sqrt{\frac{\chi_{Th}}{g\chi}}\right)^j \right.\\
&\left. I_j\left(\frac{\sqrt{g\chi\chi_{Th}}}{\sigma_n^2}\right)\right\} f_\chi(\chi)\,d\chi \Bigg\} = \\
&\frac{CA_vA_r}{4}\left\{\sum_{k=1}^{\beta_v} a_{vk}\left(\alpha_v k\right)\left(\frac{\alpha_v\beta_v}{\beta_v\gamma_v+\Omega'}\right)^{-\left(\frac{\alpha_v+k}{2}\right)}\right\} \\
&\exp\left(-\frac{\chi_{Th}}{2\sigma_n^2}\right)\times\frac{1}{2\sqrt{\pi}} \sum_{j=1}^{m-1}\left\{\left(\frac{\chi_{Th}}{g}\right)^j \sum_{k=1}^{\beta_r}\left[a_{rk}\left(\frac{1}{\xi}\right)^{\left(\frac{\alpha_r+k}{4}\right)}\right.\right. \\
&\times \sum_{q=0}^{\infty}\left(\left(\frac{\sqrt{g\chi_{Th}}}{\sigma_n^2}\right)^q \int_0^\infty \exp\left(-\frac{g\chi}{2\sigma_n^2}\right)\chi^{\frac{\alpha_r+k+2q-4j}{4}}\right. \\
&\left.\left.\left.\mathbf{S}\left[\begin{array}{c|c|c} \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} & & \\ \begin{pmatrix}1,1\\ 0,1\end{pmatrix} & \begin{matrix}-;-\\ \frac{1}{2};j,-j\\ -;b,-b\end{matrix} & \begin{pmatrix}w\sqrt{\chi}\\ x\sqrt{\chi}\end{pmatrix} \\ \begin{pmatrix}2,0\\ -2,2\end{pmatrix} & & \end{array}\right] d\chi\right)\right]\right\}
\end{aligned} \tag{23}$$

where b in (23) is equalt to $\frac{\alpha_r+k}{2}$, $w=\frac{2\sqrt{g\chi_{Th}}}{\sigma_n^2}$, and $x=\left(\frac{\alpha_r\beta_r}{\beta_r\gamma_r+\Omega'}\right)\frac{1}{\sqrt{\xi}}$.

The integral containing BMGF $\mathbf{S}[\cdot|\cdot|\cdot]$ in (23) can then readily be solved using the identity [20, eq. (2.1)].

ACKNOWLEDGMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2018R1D1A3B07049858).

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